UNIT 4

Make sure to do activity 4.8

Also do question 2 of assignment 2

# Sets

Remember a few things for this unit:

* U (Union) is also code for “OR”. This is very helpful in proofs where you have to break down set notation
* Learn what each of the “operators” for sets does (). You need to improve how your mind reads them so drawing and picturing the Venn diagrams is much easier
* {} Hardest operators: . Knowing when to use the set difference () and symmetric set difference () is VERY NB.
* Think of set compliment () as the (!) operator from C++. It basically reverses any statement it is attached to.
* DO ACTIVITIES 4.8, 4.10 and 4.12 from this unit

Try to use (‘) and () interchangeably.

x ∈ (A ∪ B)’ = x ∉ (A U B)

# Logical symbols ()

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | AND | OR | If… Then  (implication) | If and only if (Biconditional) | Its not the case that |
| Math Symbol |  |  |  |  | not |
| C++ | && | || | If, else | if | ! |
| Logical |  |  | → | ↔ | ¬ |

* This is EXTREMELY important for proofs, you need to learn how to convert operators in Maths too. (https://www.decodedscience.org/introducing-math-symbols-union-intersection/16364)

# How to do proofs (For relations, very NB!)

➊ Let be the element of each relation

➋ Separate as an element of each set in relation

➌ Convert relational operators into words

➍ Convert () and (‘) into the opposite.

Example 1 (from Activity 4.6, (b) page 58)

Prove that X – (Y ∩ W) = (X – Y) ∪ (X – W)

Left hand side

X – (Y ∩ W)

➊ Let ∈ X and ∈ (Y ∩ W)

iff ∈ X and ∈ (Y ∩ W)

➋ iff ∈ X and ∉ (Y ∩ W)

➌ iff ∈ X and ( ∉ Y and ∉ W)

iff ( ∈ X and ∉ Y) or ( ∈ X and ∉ W)

➌ iff ( ∈ X and ∉ Y) ∪ ( ∈ X and ∉ W)

iff ∈ (X - Y) ∪ (X-W)

Therefore X – (Y ∩ W) = (X – Y) ∪ (X – W) for all subsets X,Y and W of U

* The trickiest part is separating the brackets. It’s kind of like distribution. When you multiply operators with each other.

iff ∈ X and ( ∉ Y and ∉ W)

iff ( ∈ X and ∉ Y) or ( ∈ X and ∉ W)

# De Morgan’s law

If A and B are any two sets then,

* **(A ∪ B)′ = A′ ∩ B′**

The complement of A union B equals the complement of A intersected with the complement of B.

* **(A ∩ B)′ = A′ ∪ B′**

The complement of A intersected with B is equal to the complement of A union to the complement of B.

# Understanding Methods of proof

**Conjecture**

**Law**

**Hypothesis**

**Theorem**

Proof

**Proof**

**Proof**

**True statement**

**False statement**

**Proof**

**Counter Example**

**Theorem**

“**If** *condition* **then** *conclusion*”

p is condition

q is conclusion

p q

Conditional statement

Biconditional statement

``*p* if and only if *q*''

p is condition

q is conclusion

p q

Find an element of x that does not satisfy the conclusion

Test each element of the set through an exhaustive search

**Counter Example**

# Types of proofs

**Direct Proof** – Assume P, show Q

A **direct proof** is a sequence of statements which are either givens or deductions from previous statements, and whose last statement is the conclusion to be proved.

* That is to say that if p r  and r q then it follows that p q
* We start off assuming that p is true

*You need to use knowledge based on the information given. So if in the conjecture you see prime number you need to list/remember the facts about prime numbers.*

**Proof by contradiction** - Assume P and ¬Q, derive a contradiction

A **proof by contradiction** is a **indirect proof**, that establishes the [truth](https://en.wikipedia.org/wiki/Truth#Formal_theories) or [validity](https://en.wikipedia.org/wiki/Validity) of a [conjecture](https://en.wikipedia.org/wiki/Proposition)

1. Assume the given conjecture is false , that is to say assume P is false (¬ P is true)
2. ¬ P implies Q and ¬ Q
3. Since Q and ¬ Q cannot both be true, 1 is wrong and P must be true

*This is like proving stuff with the assumption that the conjecture given was false.*

**Contrapositive/Equivalent forms** - Assume ¬Q, show ¬P

A **proof by contrapositive** is formed by negating both terms and reversing the direction of inference

* Assume ¬Q, show ¬P

*This one is very easy, if you can prove one conjecture in the negative, the other conjecture will prove positive, then two negatives make a positive*